CONSISTENT SYSTEMATICS OF LEVEL DENSITY FOR MEDIUM AND HEAVY NUCLEI

O.T. Grudzevich, A.V. Ignatyuk, V.I. Plyaskin, A.V. Zelenetsky

Institute of Physics and Power Engineering Obninsk, USSR

Abstract: The systematics of experimental data on the density of neutron resonanses and low-lying levels is considered taking into account the vibrational enhancement of level density and also the shell and superfluid effects. The energy dependence of level density predicted by the generalized superfluid model is in good agreement with the data derived from the evaporation spectra analysis of different reactions.

## Introduction

The density of the excited levels is one of the most important characteristics in the statistical description of different processes related to the decay of a compound nucleus. The widely used Fermi-gas model /1,2/ fails to give an adequate consistent description of various types of experimental data on the statistical properties of nuclei, since it does not take into account the shell inhomogeneities in the single-particle level spectrum, the correlation effects of the superconductive type and coherent effects of the collective nature. The microscopic methods of analysis of these effects /3,4/ appear rather time-consuming, which duly restricts their applicability. It is, therefore, important to find a description of the level density which would take into consideration the main ideas of the theory of highly excited nuclei while remaining sufficiently simple and convenient for practical applications. The consistent phenomenological level density systematics for heavy nuclei with A > 150, where rotational effects are significant, was discussed in /5/. In the present paper we should like to extend this systematics to lighter nuclei, where the collective level density increase is determined by the vibrational excitations.

## 1. Relations of Generalised Superfluid Model

Let us discuss briefly the main components of a consistent description of the nuclear level density. The influence of the pairing effects of the superconductive type on the nuclear properties can be characterised by the correlation function  $\Delta_{\rm o}$ , which determines the even-odd nuclear mass differences and the gap value  $2\Delta_{\rm c}$  in the quasiparticle excitation spectrum of the even-even nuclei. Related to the correlation function in excited nucleus are the critical temperature  $\rm t_{cr}=0.567\Delta_{\rm o}$  of the phase transition from the superconducting state to the normal one and the critical excitation energy

$$U_{cr} = 0,472a_{cr}\Delta_o^2 - n\Delta_o , \qquad (1)$$

where n = 0,1 and 2 for the even-even, odd and odd-odd nuclei respectively. Above the critical energy the excited state level density and other statistical characteristics of nucleus can be described in terms of Fermi-gas model relations, in which we only need to introduce the effective excitation energy

$$U^* = U - 0,152a_{cr}\Delta_0^2 + n\Delta_0$$
 (2)

Below the phase transition point the relations for the thermodynamical functions are more complicated. These functions in a form convenient for practical calculations are presented in /5,6/. The shell inhomogeneities in the

The shell inhomogeneities in the single-particle spectrum lead to a certain dependence of the level density parameter a(U) on the excitation energy. However the shell effects become weaker with the increase of the excitation energy and at high energies the level density parameter can be defined by the asymptotic value

$$\widetilde{\mathbf{a}} = \alpha A + \beta A^{2/3} \tag{3}$$

The following relation was used /5/ for a phenomenological description of the energy dependence of this parameter:

$$\mathbf{a}(\mathbf{U},\mathbf{Z},\mathbf{A}) = \begin{cases} \tilde{\mathbf{a}} \left\{ 1 + \delta \mathcal{E}_{o}(\mathbf{Z},\mathbf{A}) \mathbf{f}(\mathbf{U}^{*}) / \mathbf{U}^{*} \right\}, \quad \mathbf{U} \geqslant \mathbf{U}_{c} \\ \mathbf{a}_{cr}(\mathbf{U}_{cr}\mathbf{Z},\mathbf{A}), \quad \mathbf{U} < \mathbf{U}_{cr} \end{cases}$$
(4)

where  $\delta \mathcal{E}_o$  is the shell correction in the nuclear binding energies /7/,  $f(U) = 1 - \exp(-\chi U)$  is a "universal" function describing the shell effects damping in highly excited nuclei. We kept this description for level density systematics in lighter nuclei using the value

in lighter nuclei using the value

= 0.4/ A<sup>1/3</sup>, MeV<sup>-1</sup>/8/.

To take into account the collective effects the density predicted by the superfluid model have to be multiplied by the coefficient of the vibrational enhancement of the level density

$$K_{vib}(U) = \exp(\delta S - \delta U/t),$$
 (5)

where dS and dU are entropy and excitation energy changes resulting from the Bose branch of collective excitations.

These changes are described by the relations:

$$dS = \sum_{i} (2\lambda_{i}+1) \left[ (1+\overline{n}_{i}) \ln(1+\overline{n}_{i}) - \overline{n}_{i} \ln(\overline{n}_{i}) \right],$$

$$U = \sum_{i} (2\lambda_{i}+1) \omega_{i} \overline{n}_{i}, \qquad (6)$$

where  $\omega_i$  are the energies,  $\lambda_i$  are the degrees of degeneracy and  $n_i$  are the occupation numbers of vibrational excitations at a given temperature t. If the ideal Bose-gas relations are used for the occupation numbers, then the expression (5) will be equivalent to adiabatic addition of vibrational excitation spectrum to all possible quasi-particle excitations of the nucleus /9/.

The rigorous approach to determining occupation numbers would have required solving a complicated many-particle problem of interacting Bose and Fermi excitations. We did not aim at solving such a problem, but only took the simplest approximation for the occupation numbers

$$\bar{n}_i = \exp(-\gamma_i/2\omega_i) \left[\exp(-\omega_i/t)-1\right]^{-1}, \quad (7)$$

where  $\chi$  is the width damping of Bose excitations. As  $\chi=0$  this approximation gives the ideal gas relations, but it provides a reduction of  $K_{vibr}$  as  $\chi$  increases. One should expect the collective excitation damping in nuclei to be similar to a certain extent to the zero-sound damping in Fermi liquid /10/ which is described by

$$\chi = c(\omega_i^2 + 4\pi^2 t^2)$$
 (8)

An estimation for  $C = 0.05 \text{ A}^{1/3}$  MeV can be obtained from the observed widths of the giant quadrupole isoscalar resonances.

In calculating level densities we have used experimental values for energies of the first 2  $^+$  levels of the eveneven nuclei /11/ and extrapolations of these values for the neighbouring odd and odd-odd nuclei. The average value of the observed energies  $\omega_{3^-} = 50 \text{ A}^{-2/3} \text{MeV}^{-1}$  was used for octupole excitations whose influence is much weaker than that of quadrupole ones.

The experimental values for shell corrections  $\delta \mathcal{E}_o$  were taken from /7/ and the values for  $\delta \mathcal{E}_o$  were taken from /7/ and the values for  $\delta \mathcal{E}_o$  were taken from /7/ and  $\beta = 0,115 \text{ MeV}^{-1}$  obtained in /5/ were used for the asymptotic level density parameters (3). The nuclear correlation functions were assumed to be  $\Delta_o = 12/A^{-1/2}$ . The coefficient  $C = 0,0075 \text{ A}^{1/3} \text{ MeV}^{-1}$  characterizing the effective decrease of vibrational enhancement of level density for highly excited nuclei was determined from an optimal description of all experimental data on neutron resonance densities /12/.

These optimal parameters do not necesserily guarantee an exact agreement with the experimental data for individual nucleus. Such an agreement, however,

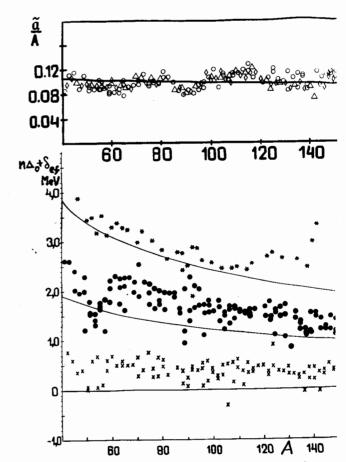


Fig. 1 Systematics of the ratio of the asymptotic value of the level density parameter  $\tilde{a}$  to the mass number (top) and the value of the effective energy shift  $n\Delta_{o} + \delta_{eff}$  (bottom).

is often required for calculating neutron evaporation spectra and excitation functions of different reactions for particular nuclei. Using the above mentioned approach we found the set of parameters a and Oeff, which describe the neutron resonance densities /12/ and the cumulative number of the observed low-lying levels /11/ for each individual nucleus. The parameters obtained are shown in Fig. 1. Individual parameters fluctuate and such fluctuations reflects foremost the simplifications based on the replacement of correlation functions for neutrons and protons by an average correlation function  $\Delta_o$ . Correlation functions values for nuclei with the magic number of protons or neutrons should be considerably lower then average ones and these differences are left in individual parameters.

## 3. Testing Systematics of Level Density

To test the existing systematics it is necessary to extend the collection of experiments from which one could obtain information on nuclear level density. The methods for deriving the level density from nonresonance data are discribed

in /13/. The paper discussed the main problems in analysing experimental spectra of particles emission and short-coming of most the publication dealing with such analysis. The author shows that it is expedient to derive from experimental spectra directly absolute values rather than the model parameters for the level density. To check the results obtained one could use the discription of the hard part of the spectra in the range of well-identified residual nucleus levels. It stands to reason that one has to make sure that the spectral range involved is not distorted by any nonstatistical processes. The (p,n),  $(\alpha,n)$  and  $(\alpha,p)$  reaction spectra with incident particles under 15 MeV appear to be the best in that respect. It is analysing these spectra that our testing was based focused on.

Let us take the results of a neutron spectrum analysis for the reaction  $^{65}$ Cu(p,n) $^{65}$ Zn as a typical example. The energy dependence of the level density obtained from the experiment by the method suggested in /13/ is shown in fig. 2a. When excitation energy is low the level density obtained is in good agreement with the discrete low-lying level data observed directly. Fig. 2a also shows two phenomenological descriptions of level density: in the back shift Fermi-gas model and in the generalized superfluid model. The parameters in both of the models have been determi-ned by conditions of describing the experimental data on low-lying levels and neutron resonance density. That is why a more or less significant disagreement between the models can be found in the intermediate range only. It can be seen that the neutron spectrum data agree with the superfluid model's prediction better than with that of the Fermi-gas

The levels density parameter a in the Fermi-gas model shown on Fig.2a is a = 7,07 MeV $^{-1}$  and the excitation energy shift is  $\delta$  = -1,01 MeV /2/. On the other hand the spectrum analysis within the

Fermi-gas model in /14/ gave a=10,7 MeV and  $\delta$  = -0,4 MeV. The reason why the parameters differ so much will become clear if we consider the level density predicted by the generalized superfluid model. The energy dependence of the level density proves to be different above and below the critical energy of the phase transition from the superconductive state to the normal state. Below the critical energy the energy changes of the level density are determined to a considerable extend by the correlation function value  $\Delta_0$  rather than by the level density parameter a. If the greater portion of the data from evaporation spectra falls below the critical energy then any attempt at describing the observed energy dependence of the level density in terms of the Fermi-gas model will inevitably result to the considerable overestimation of the level density parame-

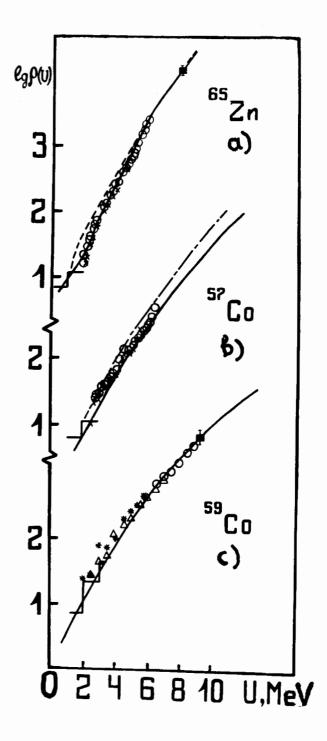


Fig.2 The energy dependence of the level densities derived from the low-lying levels (histograms), the neutron resonance densities ( ) and the evaporation spectra analysis (the rest signs). The solid curves shows the level density description in the generalized superfluid model and the dashed curves corresponds to the back-shift Fermi-gas model.

ters. However, when a substantial portion of the spectrum analysis results is above the critical energy, then the inaccuracy of simple Fermi-gas level density parametrization proves to be less significant and the data obtained from the evaporation spectra will be close to the parameter values based on the neutron resonance density description. The results of experimental data analysis are also affected by superfluid properties being weakened for nearmagic nuclei, which makes the difference between the Fermi-gas model's predictions and those of the generalized superfluid model less significant. A set of data for the <sup>57</sup>Co nucleus shown in Fig. 2b de-

monstrates it. The energy dependence of level density for both of the models proves to be very similar, but there are differences in absolute value predictions for level densities because of the lack of data on neutron resonance den-

The great interest for the study of nuclear level density are examinations of particle spectra from different reactions leading to the same residual nucleus. The analysis of those data gives additional criteria for checking the hypothesis of a statistical reaction mechanism and the correctness of the transmission coefficient chosen for the different decay channels of the compound nucleus. Fig.2c shows the level density

data for  $^{59}$ Co, obtained by analysing the  $^{56}$ Fe( $\alpha$ ,p),  $^{59}$ Co( $\alpha$ , $\alpha$ ) and  $^{62}$ Ni(p, $\alpha$ ) reaction spectra /15/. The results of spectrum analysis for different particles are fairly closely matched. A certain spread of the  $(\mathscr{A},\mathscr{L})$  and  $(p,\mathscr{L})$  data is accounted for by fluctuations of experimental points in the corresponding  $\alpha$ -particle spectra /15/. The energy dependence of the level density, obtained from the spectrum analysis is in good agreement with the predictions of the systematics based on the data for lowlying nuclear levels and neutron resonance density. It should be noted that there are also neutron and proton inelastic spectra for 59 co target in addition to the above mentioned data. Analysing those spectra, however, does not help much in the study of the level den-

The analysis of evaporation spectra of different reactions has shown that parameters of the generalized superfluid model, based on the experimental information on neutron resonance density and low-lying nuclear levels are supported also by the data derived from the observed spectra. The discussion in the present paper is restricted to the analysis of the spectra for nuclei from the Fe region. An expansion of the method developed to cover a wider range of nuclei may be of interest both for getting a

better knowledge of the level density

sity because of the direct and preequilibrium nucleon emission mechanisms' contributing to the experimental spec-

tra considerably.

for excited nuclei and for an increased reliability of the description of energy dependence of the level density in currently used theoretical models.

The necessity of using more rigorous, but inevitably more complicated, models than that of the Fermi-gas for the level density description and systematics now appears to be almost evident. The complications of the analysis are justified by the consistency parameters characterising the diverse experimental information on the statistical properties of nuclei. We hope that the approach suggested in present paper may prove helpful for practical calculations of the level densities in a wide range of excitation energies and mass numbers.

## REFERENCES

- 1. A.V. Malyshev: Level density and Structure of atomic nuclei (Russian).
- M., Atomizdat, 1969

  2. W. Dilg et al: Nucl. Phys. A217, 265(1973)
- 3. A.I. Vdovin et al: Phys. elem. part. and nucl. 7, 952(1976)
  4. A.V. Ignatyuk: Statistical Properti-
- es of Excited Atomic Nuclei (Russian)
- M., Energoatomizdat (1983)

  5. A.V. Ignatyuk et al: Sov. Nucl.
  Phys. 29, 875(1978)

  6. A.V. Ignatyuk et al: Izv. Acad. Sci.
  USSR, ser. phys. 37, 1974(1973)

  7. W.D. Myers: Droplet Model of Atomic nuclei. IFP/Plenum, 1977

  8. V.S. Rememurthy et al: In: IAEA Me-
- 8. V.S. Ramamurthy et al: In: IAEA Me-eting on Basic and Applied Problems eting on Basic and Applied Problems of Nuclear Level Density. N.Y.: INDC-092/L, 187(1983)

  9. A. Bohr, B. Mottelson: Nuclear Structure, N.Y., Benjamin, 2 (1975)

  10. L.D. Landau: Sov. JETP, 32, 59(1957)

  11. C.M. Lederer, Y.S. Shirley: Tables of Isotopes. N.Y.: 7th, ed., (1979)

  12. T.S. Belanova et al: Radiactive Capture of Neutrons (Russian) M. Energy

- ture of Neutrons (Russian) M., Ener-gaatomizdat (1986)
- 13. H. Vonach: In: IAEA Advisory Group Meeting on Basic and Applied Prob-lems of Nuclear Level Densities
- INDC(USA)-092/L, 187(1983)
  14. G.N. Lovchikova et al: In: Neutron Induced Reactions, Bratislava, VEDA, 82(1986)
- 15. A.I. Kennedy et al: Phys. Rev. <u>C5</u>, 500(1972)